

Summer Work for A level Mathematics

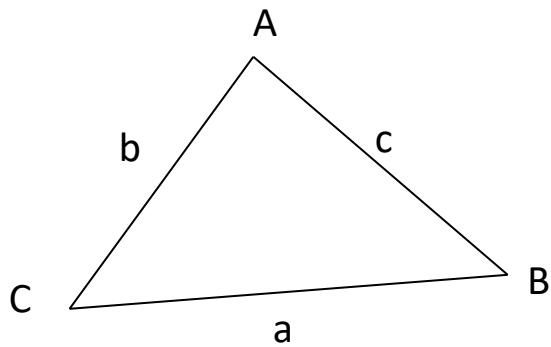
A-level mathematics assumes you know GCSE! Very few people master absolutely everything in GCSE maths but the questions here are to practise the skills which are really essential. If there are other topics in GCSE that you know you struggled with, practise them as well because they will probably be needed. (vectors and circle theorems are two that come to mind!)

- Complete the questions on the following pages to help you assess your skills.
- Answers are provided for this task.
- When you are confident tackling questions on the above topics complete the questions on the Test Page.
- Hand it to your Maths teacher in the first week of the Autumn Term.

Checklist

	Topic	Confident	Comfortable	Need to work on
1	Types of number			
2	Fractions			
3	Laws of indices			
4	Factorising			
5	Algebraic fractions			
6	Changing the subject			
7	Quadratic equations			
8	Simultaneous Equations			
9	Gradient			
10	$y=mx+c$			
11	Distance between two points			
12	Proportion			
13	Inverse proportion			
14	Sine rule and cosine rule			

Useful Formulas



Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of a triangle

$$\text{area} = \frac{1}{2}ab\sin C$$

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1. Types of Numbers

i. Are these expressions rational or irrational? Explain your answers.

a. $\sqrt{2}$

b. 0.236849

c. $\sqrt{64}$

d. 2π

e. $(\sqrt{5})^2$

f. $\sqrt{8} \times \sqrt{2}$

ii. Which of question 1 parts, a, c, e and f are surd form?

iii. True or false?

a. All integers are rational.

b. All surds are real numbers.

2. Fractions

i. $\frac{3}{4} + \frac{1}{3}$

ii. $1\frac{5}{6} + 2\frac{1}{2}$

iii. $5\frac{1}{3} - 1\frac{1}{2}$

iv. $\frac{3}{4}$ of $\frac{7}{8}$

v. $2\frac{2}{3} \times \frac{1}{4}$

vi. $1\frac{3}{4} \div \frac{5}{6}$

vii. $5\frac{1}{3} \div 2\frac{1}{4}$

3. Laws of Indices

Express the following in their simplest form:

i. $9^{\frac{1}{2}}$

ii. $81^{\frac{1}{4}}$

iii. $64^{\frac{1}{6}}$

iv. 27^0

v. $4^{\frac{3}{2}}$

vi. $64^{\frac{5}{6}}$

vii. $64^{\frac{-2}{3}}$

viii. $b^5 \times b^6$

ix. $g^7 \div g^3$

x. $y^{10} \times y^2 \div y^5$

xi. $\left(\frac{125}{8}\right)^{\frac{1}{3}}$

xii. $\left(\frac{16}{9}\right)^{\frac{-3}{2}}$

4. Factorising

Factorise

i. $9x - 21z$

ii. $20x^2 - 4x$

iii. $8x^2y + 28xy^2$

iv. $3\pi a^2 + 4\pi ab$

v. $6 + 2x^2$

vi. $y^3 + 3y^2 - y$

vii. $4x(2x + 3) - 3(2x + 3)$

viii. $5x^2(x - 1) - 2x(x - 1)$

ix. $x^2 - 9$

x. $4x^2 - 16$

xi. $p^2 - q^2$

iv. $9m^2 - 25n^2$

5. Algebraic Fractions

i. Cancel these fractions as far as possible:

a. $\frac{3x^2}{7x}$

b. $\frac{8x^2(x+3)}{4x}$

c. $\frac{8x-16}{2x-4}$

ii. Express as a single fraction:

a. $3 + \frac{2}{x}$

b. $\frac{1}{x+1} - \frac{3}{x-2}$

c. $\frac{a}{b} - \frac{2a}{3b}$

iii.. Simplify these expressions:

a. $\frac{3x+9}{4} \times \frac{x}{3(x+3)}$

b. $\frac{x+3}{x^2} \times \frac{x}{4}$

c. $\frac{x(x-3)}{3} \div \frac{x-3}{x}$

d. $12x(x+2) \div \frac{3x+6}{x}$

6. Changing the Subject of a Formulae

i. $x = 5y^2$. Find y in terms of x , if y is a positive number.

ii. Express a in terms of b , give that: $b(a+2) = 4$

iii. Make C the subject of the formula: $F = \frac{9}{5}C + 32$

iv. Make w the subject of the formula: $v = \frac{w+2}{w+3}$

v. Make t the subject of the formula: $s = \frac{\sqrt{(t+u)}}{u}$

vi. Make x the subject of the formula: $y = 3\sqrt{x \div 2}$

7. Quadratic Equations

Solve the following quadratic equations:

i. $x^2 + 3x + 2 = 0$

ii. $x^2 + 8x + 7 = 0$

iii.. $x^2 + 4x - 12 = 0$

iv. $x^2 - 4x - 12 = 0$

v. $x^2 - 7x + 10 = 0$

vi. $x^2 - 14x + 40 = 0$

vii. $2x^2 + 9x + 9 = 0$

viii. $5x^2 + 13x + 6 = 0$

ix. $2x^2 - x - 10 = 0$

8. Simultaneous Equations

i. Solve these simultaneous equations, by a method of elimination:

a. $5p + 2q = -30$
 $3p + 4q = -32$

b. $7x - 3y = 48$
 $x + 0.5y = 5$

c. $5x + 3y = 17$
 $4x + 10y = 25$

ii. Solve these simultaneous equations, by a method of substitution:

a. $x = 2y + 1$
 $3x - 4y = 7$

b. $a - 3b = 11$
 $5a + 2b = 4$

c. $3x + y = 7$
 $x - 2.5y = 8$

iii. In a quiz, 1 correct answer and 3 incorrect answers scores 6 points, whilst 2 correct and 4 incorrect scores 16 points.

a. What is the value of a correct answer?

b. How many points are deducted for an incorrect answer?

9. Gradient

Find the gradients of the lines which pass through the following points:

i. (5, 6) and (9, 15)

ii. (2, 12) and (4, 1)

iii. (-6, -2) and (3, 4)

iv. (-4, 8) and (10, -3)

10. $y = mx + c$

i. Find the equation of the line with gradient 3 and y-intercept 5.

ii. Find the equation of the line which has y-intercept -2 and gradient 4.

iii. Find the equation of the line which has gradient -2 and which passes through the point (0, 6).

11. Distance Between 2 Points

i. Find the distance between points P(2, 6) and Q(5, 14).

ii. Find the lengths of each of the sides of the triangle with vertices at points A(3, 5), B(7, 2) and C(-1, 3) respectively.

iii. Find the lengths of the sides of triangle PQR which has vertices at points P(-4, 2), point Q(-1, 6) and point R(3, 3). What kind of triangle is this?

12. Proportion

- i. Given that y is directly proportional to x^3 and that $y = 9$ when $x = 3$, find y in terms of x .
- ii. Given that a is directly proportional to b^2 and that $a = 4.32$ when $b = 1.20$, find a when $b = 2.5$.

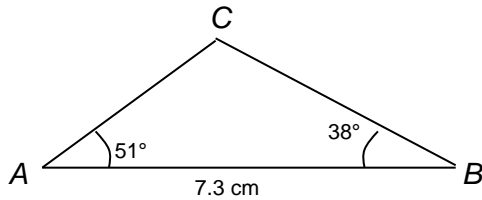
13. Inverse Proportion

- i. Given that y is indirectly proportional to x^3 and that $y = 0.25$ when $x = 2$, find y in terms of x .
- ii. Given that p is inversely proportional to \sqrt{q} and that $p = 2$ when $q = 25$, find p when $q = 36$.

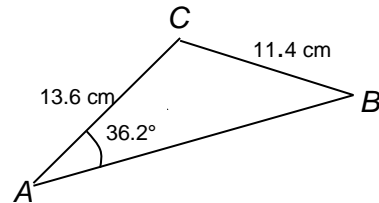
14. Sine Rule and Cosine Rule

Calculate the lengths of the unknown sides and angles

i.

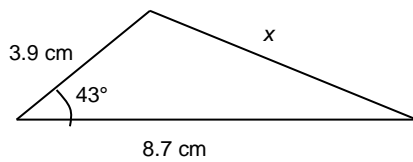


ii.

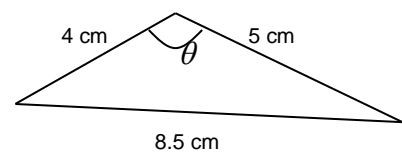


Calculate the length x and angle θ

iii..



iv.



Summer Work – Answers

1. Types of Numbers

- i. a. $\sqrt{2}$ irrational – non-repeating, never ending decimal
- b. Rational $0.236849 = \frac{236849}{1000000}$ c. Rational $\sqrt{64} = 8 = \frac{8}{1}$
- d. Irrational $6.28318\dots$, non-repeating, never ending decimal.
- e. Rational $(\sqrt{5})^2=5$ f. Rational 4
- ii. a. Surd because irrational. b. Not surd as 8 is rational.
- c. Not surd as 5 is rational. f. Not surd as 4 is rational.
- iii. a. True b. True

2. Fractions

- i. $1^{1/12}$ ii. $4^{1/3}$ iii. $3^{5/6}$ iv. $2^{1/32}$
- v. $2^{2/3}$ vi. $2^{1/10}$ vii. $2^{10/27}$

3. Laws of Indices

- i. 3 ii. 3 iii. 2 iv. 1
- v. 8 vi. 32 vii. $\frac{1}{16}$ viii. b^{11}
- ix. g^4 x. y^7 xi. $\frac{5}{2}$ xii. $\frac{27}{64}$

4. Factorise

- i. $3(3x - 7z)$ ii. $4x(5x-1)$ iii. $4xy(2x+7y)$
- iv. $\pi a(3a + 4b)$ v. $2(3+x^2)$ vi. $y(y^2 + 3y - 1)$
- vii. $(2x+3)(4x-3)$ viii. $x(x-1)(5x-2)$ ix. $(x-3)(x+3)$
- x. $(2x-4)(2x+4)$ xi. $(p+q)(p-q)$ xii. $(3m+5n)(3m-5n)$

5. Algebraic Fractions

- i. a. $\frac{3x}{7}$ b. $2x(x+3)$ c. 4
- ii. a. $\frac{3x+2}{x}$ b. $\frac{-2x-5}{(x+1)(x-2)}$ c. $\frac{a}{3b}$
- iii. a. $\frac{x}{4}$ b. $\frac{x+3}{4x}$ c. $\frac{x^2}{3}$ d. $4x^2$

6. Changing the Subject

i. $y = \sqrt{\frac{x}{5}}$ ii. $a = \frac{4-2b}{b}$ iii. $c = \frac{5}{9}(F-32)$ or $\frac{5F-32}{9}$

iv. $w = \frac{2-3v}{v-1}$ v. $t = u(s^2u-1)$ or $s^2u^2 - u$ vi. $x = \frac{2}{9}y^2$

7. Quadratic Equations

i. $x = -2$ or $x = -1$ ii. $x = -7$ or $x = -1$ iii. $x = 2$ or $x = -6$

iv. $x = -2$ or $x = 6$ v. $x = 2$ or $x = 5$ vi. $x = 4$ or $x = 10$

vii. $x = -\frac{3}{2}$ or -3 viii. $x = -\frac{3}{5}$ or -2 ix. $x = \frac{5}{2}$ or -2

8. Simultaneous Equations

i. a. $p = -4$ $q = -5$ b. $x = 6$ $y = -2$ c. $x = 2.5$, $y = 1.5$

ii. a. $x = 5$ $y = 2$ b. $a = 2$ $b = -3$ c. $x = 3$ $y = -2$

iii. a. 12 points b. 2 points are deducted.

9. Gradient

i. $\frac{9}{4}$ ii. $-\frac{11}{2}$ iii. $\frac{2}{3}$ iv. $-\frac{11}{14}$

10. $y = mx + c$

i. $y = 3x + 5$ ii. $y = 4x - 2$ iii. $y = -2x + 6$

11. Distance Between 2 Points

i. $\sqrt{73}$ ii. $\sqrt{65} = BC$
 $\sqrt{20} = AC$
 $5 = AB$

iii. $PQ = 5$ $PR = \sqrt{50}$
 $QR = 5$
Since $PQ = QR$
 $\triangle PQR$ is isosceles

12. Proportion

i. $y = \frac{1}{3}x^3$ ii. $a = 3b^2$ so $b = 2.5$ when $a = 18.75$

13. Inverse Proportion

i. $y = \frac{2}{x^3}$ ii. $p = \frac{10}{\sqrt{q}}$ when $q = 36$ $p = \frac{5}{3}$

14. Sine Rule and Cosine Rule

i. $AC = 4.5$ cm $BC = 5.7$ cm $\hat{C} = 91$
 $B = 44.8^\circ$ $C = 99^\circ$ $AB = 19.1$ cm or $B = 135.2^\circ$ $C = 8.6^\circ$ $AB = 2.9$ cm

ii. 6.42 cm iv. 141.4°

Name _____

TEST PAGE – A level Mathematics

(The following section is to be completed and handed in to your Maths teacher in the first week of term.)

Without the aid of a calculator, find the value of each of the following:

1. $144^{\frac{1}{2}}$ 2. $25^{-\frac{1}{2}}$ 3. $\left(\frac{27}{64}\right)^{\frac{2}{3}}$ *

(* this is more difficult and will be covered again in the course)

Simplify (where necessary) and factorise these expressions:

4. $15xy + 25x^2$ 5. $16xy - 7x(y + 2) - 4x$

6. $3x(5x - 4) - 7(5x - 4)$

Use cancellation to reduce these expressions to their simplest forms:

7. $\frac{4x+8}{3x+6}$ 8. $\frac{5x^3 - 15x^2}{2x - 6}$

Make x the subject of each of these formulae:

9. $y = 16ax^2$ 10. $y = \sqrt{4 - x^2}$

Factorise these quadratic expressions:

11. $x^2 - 5x - 24$ 12. $3x^2 + x - 24$

Solve these quadratic equations:

13. $x^2 - 9x + 20 = 0$ 14. $6x^2 + 7x - 5 = 0$

Use the method of substitution to solve each pair of simultaneous equations:
(if you have not covered substitution it would be helpful for you to read up on it.
Otherwise use the method of elimination)

15. $x = y - 1$
 $2x + 3y = 18$

16. $2x + y = 7$
 $3x - 2y = 21$

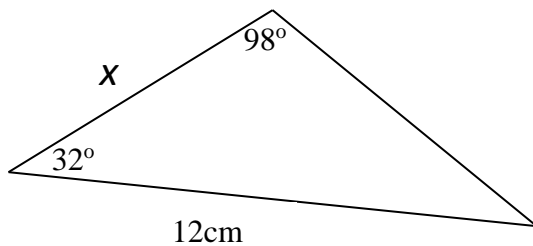
Find the equation of the line described in each of the following cases:

17. passes through (3,2) with gradient 2

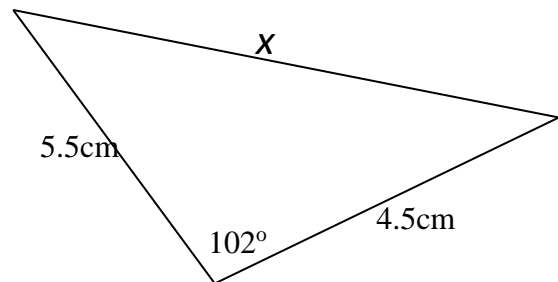
18. passes through the points (1, 2) and (7, 5)

19. Find the value of x for each of these questions:

a



b



c

